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A Study on Skills and Methods on Teaching of Transformation Geometry in Secondary Schools: A Case Study of Selected Secondary School in Chongwe District

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Abstract- This study investigated the skills and methods used in the teaching of transformation geometry in the Zambian curriculum, with particular reference to Mulalika Secondary School in Chongwe District. The study was motivated by the observation that learners often experience difficulties in understanding and applying transformation concepts such as translation, reflection, rotation and enlargement. The study aimed to identify the teaching skills employed by mathematics teachers, examine the methods used in teaching transformation geometry, establish challenges faced by teachers and learners, and suggest strategies for improving the teaching and learning of transformation geometry. A descriptive case study research design was employed. The study targeted mathematics teachers and learners at Mulalika Secondary School. Data were collected using interviews and classroom observation. Quantitative data were analyzed using frequencies and percentages, while qualitative data were analyzed thematically according to the major themes emerging from the responses. The findings indicated that teachers used a variety of teaching methods, including demonstration, discussion, question-and-answer, group work, guided discovery and use of practical activities. However, the effectiveness of these methods was affected by challenges such as inadequate teaching and learning materials, limited use of Information and Communication Technology (ICT), large classes, insufficient time for practical activities and learners' difficulties in visualizing geometric transformations. The study further found that effective teaching required skills such as clear explanation, use of diagrams and practical demonstrations, questioning, classroom management, learner engagement and the ability to relate transformation geometry to real-life situations. The study concludes that the effective teaching of transformation geometry depends on the appropriate combination of learner-centered teaching methods and relevant pedagogical skills. It recommends that mathematics teachers should make greater use of practical activities, geometric instruments, locally available materials, ICT and collaborative learning approaches. It also recommends continuous professional development for teachers and improved provision of teaching and learning resources to enhance learners' understanding and achievement in transformation geometry under the Zambian curriculum.

Keywords- transformation geometry, teaching skills, teaching methods, mathematics education, Zambian curriculum, secondary school.

I. Introduction

In all aspects of human endeavors, mathematics plays an important role which cannot be given a blind eye. The row mathematics plays the entire human race can be seen through its branches such as calculus, algebra, statistics dimension but they feel. In this chapter will take us through the background of the study, statement problem, purpose of the study, objectives of the study, research questions, significance of the study, scope of the study and definition of the study.

1. Background of the study

Mathematics is one of the core subjects in the Zambian education system because it equips learners with logical reasoning, problem-solving, critical thinking, and analytical skills. Under the Zambian Competence-Based Curriculum, mathematics is intended to develop learners who can apply mathematical knowledge and skills to solve real-life problems. Geometry is one of the major strands in the secondary school mathematics curriculum, with transformation geometry being an important topic taught from junior secondary school.

Transformation geometry involves the study of changes in the position, orientation, and size of geometric figures. The main transformations include translation, reflection, rotation, and enlargement. These concepts help learners understand spatial relationships, symmetry, coordinate geometry, and mathematical reasoning. Transformation geometry also provides a foundation for advanced mathematical topics such as vectors, matrices, trigonometry, engineering drawing, architecture, and computer graphics.

Despite its importance, transformation geometry remains one of the most challenging topics for many learners in Zambian secondary schools. National examination reports have consistently shown poor learner performance in geometry-related questions.

Many learners experience difficulties visualizing transformations, interpreting diagrams, and applying mathematical principles to solve transformation problems.

Geometry plays an important role in the primary, secondary and higher mathematics curriculum in Zambia and other countries of the world. It provides a rich source of visualization for understanding arithmetical, algebraic, and statistical concepts. Volderman (1998) stated that, "geometry provides a more complete appreciation of the world we live in.

For example, geometry appears naturally in the structure of the solar system, and the 3-Dimensional geometric shapes". At secondary school level, geometry is part of the whole Mathematics curriculum. This means that, geometry is just taught as part of the whole mathematics syllabus in components comprising selected geometry topics. Some of the topics which are part of secondary school geometry include: shapes and symmetry, menstruation, transformational geometry, earth geometry and trigonometry.

One major goal of secondary school geometry is the development of mathematical reasoning abilities and the promotion of a deeper awareness of the real world. To help learners achieve this goal, the Zambian secondary school mathematics syllabus suggests that reasoning about shapes should use coordinate and transformation techniques as well as the traditional synthetic techniques such as flip, turn and sliding of an object. However, over the years this has encountered some challenges in its implementation. Among the challenges include, learners are incapable of dealing with the demands of traditional learning of high school geometry

The school mathematics resulted from the confluence of two traditions. The first rooted in Babylonia astronomy, Egyptian earth measurement and ancient commerce, is mathematics as a reckoning, as a tool required for everyday life. The second tradition is rooted in Greek geometry and medieval algebra, is mathematics as reasoning, as one of liberal art whose mastery marks an educated person. In this tradition, mathematics offers aesthetics satisfaction as well as a means of developing the mind capacity for abstract thought. Every society attempts to pass to its children the language and skills it has acquired or developed for dealing with numerical and spatial problems. When schools are organized to give children grounding in their culture or to achieve their desires, this practical sort of mathematics is what appears in the curriculum.

According to Marut (1999; P.35), mathematics arose from the need for a system for counting and calculating areas of surfaces and volumes of objects, but it has over the centuries become less concerned with practical matters and has turned instead to logic and pure intellectual speculation. He further noted that mathematics, the mother and language of all sciences has been defined by many as the science of number and space. He furthermore noted that the great achievement of technology in all its forms, which deeply influenced the life of every human being, has led to a widespread recognition of the importance of mathematics and thus interest in mathematics has therefore grown steadily.

Davies (2001) noted that without mathematics, there will be no development for the scientists as mathematics is the guarantee for precision and objectivity. Modern science and technology derive their inspiration and initiation from the philosophy that affirmed the mathematical design of nature. Pilant (2009) noted that mathematics allows scientists to communicate ideas using universally accepted terminology and that it is truly the language of science. He further noted that many technologies that we take for granted today developed from the results of mathematical research. These include fiber-optic cables that carry our telephone messages, Doppler radar images used in weather forecasting, fuel-efficient jets airplanes and streamline automobile and medical imaging processes.

Furthermore, geometry is the mathematical study of shapes and sizes of figures termed plane geometry, when plane figures are involved and analytical geometry when algebra and coordinates (numbers) are applied to geometric problems. Additionally, geometry is designated Euclidean when axioms of Euclid form the basis of the system-particularly parallel postulates, namely that two parallel lines do not intersect and non-Euclidean geometry when different sets of postulates are used to develop a consistent system. Pilant (2009) noted that geometry is the branch of mathematics that deals with the properties of space. That student in high school studies plane geometry-the geometry of flat surfaces-and may move onto solid geometry, the geometry of three-dimensional surfaces. He further noted that geometry has many more fields, including the study of spaces with four or more dimensions.

According to Kline (1979), the derivation of the term, geometry, is an accurate description of the works of the early geometers who were concerned with such problems of measuring the size of fields and laying out accurate right angles for corners of buildings. That this empirical geometry that flourished in ancient Egypt was refined and systematized by the Greeks. He further noted that Pythagoras laid the corner stone of scientific geometry showing that the various arbitrary and unconnected laws of empirical geometry could be proved to follow as logical conclusion from a limited number of axioms or postulates. He further noted that these postulates were taken by Pythagoras and his successors to be self-evident truth but in modern mathematical thinking they are considered merely as a group of convenient but arbitrary assumptions. According to Hushkowtiz, Bruckhenier and Vinner (1987), a basic knowledge of geometric concepts, their attributes and simple relations is fundamental for children to interact effectively with their environment as well as for them to enter into a formal study of geometry, itself, and other areas like science and engineering. Also Fey (1991) noted that all geometry instruction is to foster intellectual formation, that is, students should come to know what geometrical thinking is, what geometry is, what it studies and how it devises its method to do this study. He noted that geometrical thinking should not be identified as logical thinking for the latter is the domain of all mathematics.

He further noted that the goal of geometry is to transmit important information about space that has been provided in the past and appear to be necessary in the years to come. That this necessity applies not only to preparing for further study of mathematics but for applying geometric knowledge to specific everyday affairs. He furthermore noted that knowledge of geometry is to develop skills in geometric problem-solving that is techniques by which one may find answers to unknown situations through building of geometrical models of physical and behavioral theories or by using geometry as a means of explanation.

According to Marjoram (1974), researches into how children learn mathematics indicated that learning based on intuition and experiences of the children can enhance retention and application. Also Fremont (1969) noted that these research findings arrived at differing points of view regarding learning process but has established common characteristics as well. That the direction of learning moves from concrete, physical experience with complete freedom to project hypotheses and test them in given situations to the more logical, analytical abstract ordering of what has been done. Furthermore, those teachers who are intended upon building mathematics experience for children cannot ignore these works if they are to help each of their students to experience the joy and satisfaction that are all often missed in mathematics classroom. Lassa (1998; P.6) opined that the children in secondary schools should be made to have pleasure in learning mathematics. He noted that on the whole it used to be dreadful business, and no attempt was made to select subject matters that reflected their culture and be related to their future profession. He further noted that our ideas about the subject depended very much upon the ways in which we study mathematics in the schools and colleges. That some of us experienced only rote learning where teachers showed us on backboard, how to work out a particular problem, then we had to do a number of examples in the same way often without understanding what we were doing, but always trying very hard to get the right answer.

However, we did not find it easy to do examples that were not exactly like those we had been shown. And that others of course were fortunate, that teachers helped them to understand what they were doing, by providing practical activities and encouraging them to find out as much as possible for themselves. Thus, that they became confident in dealing with questions they had not seen before. He furthermore noted that with the demand for more mathematical knowledge of all professions and for coping with life there is need to change the method of teaching mathematics in the secondary school.

Timku (2004; P.111) noted that generally speaking, there are two extreme methods of teaching geometry, which are intuitive approach and the formal/axiomatic approach. He opined that for any topic of instruction, a place can be found for it between these two extremes. He further noted that recommendations are converging on the position that emphasis should be laid more on practical, intuitive numerical application strategies rather than relying on the previous strategies which emphasized memorization of book theorems.

However, Fey (1991; P.839) observed that despite the obvious importance of geometry to wide range of important real world problems and the strong traditional belief that geometry is ideal vehicle for teaching logical reasoning, the place of geometry in contemporary school is neither satisfactory nor settled Lassa (1998; p.4) observed that mathematics in our schools, colleges and universities is in a sorry state. That there have been so much concern and outcry from many quarters about the poor performance of students in mathematics at Senior School Certificate Examination (SSCE) and universities. He furthermore observed that hardly is there any day when issues relating to education are not reported in our daily papers and that in most cases, the issue discussed relate to bad performance of students in mathematics and its attendant consequences in our quest for technological development. He noted that we need to face this squarely and increase demand for mathematics as we enter the twenty-first century.

Fletcher (1976) noted that the first contact pupils in primary schools have with the study of geometry is through motion geometry`: when a shape or an object is moved from one position to another and the motion can be thought of in terms of separate processes, under the guide of the teacher. The movements, she termed transformation and the separate processes involved include turning (rotation), sliding (translation) and mirroring (reflection). She noted that this approach to teaching geometry encourages the pupils to develop geometric thinking, explore and discover geometric ideas to the best of their ability and maturity and eventually aided them in problem solving and stimulated their interest in geometry.

Marjoram (1974) noted that the mostly neglected group of missed opportunities in secondary schools is the development of the study of geometry through the basic transformations of reflection, rotation, translation and enlargement. Bishop (1991;P.860) noted that concerning the methods of teaching geometry, which transformation approach allows, that it can be seen that ideas of rotation and reflection are all intuitively familiar to students. That many actions of body operate on objects in our environment by means of rotation, for example throwing and twisting; and mirrors, windows and water give reflection; sliding and balancing give translation. That this appeals to intuition lays the foundation for geometry as empirical study in schools.

He further noted that actual objects can be transformed as can draw shapes at later stages. That many paper-folding activities can be developed once connections are made with aesthetic ideas of pattern, symmetry and balance. Line symmetry can be approached through the practical methods of folding. That this reinforces the fact that a line of symmetry divides a shape or an object into equal halves each fitting exactly onto the other and can be useful in treatment of similarity in geometry. Tessellation can be looked at and the ideas of congruence are introduced in this framework. Students can be encouraged to look at tessellation in analyzed. Combinations of transformations can be explored and suitable axiomatic treatment proposed linking geometry with other branches of mathematics (Wynne, 1977).

The basic aim is to encourage students to approach the study of geometry in a spirit of discovery and adventure for them to understand to the fullest extent consistent with their ability and maturity how mathematical facts are discovered and how mathematical concepts are formulated and can be generalized and extended. In fact, the students should be led to develop mathematical understanding and power. For example Archimedes in attempting to find the formula for the area of a segment of a parabola, imagined this area placed at one end of a level arm and the other end the rectangle circumscribing the parabola. He then undertook to determine where the fulcrum must be placed so that the figure may be in equilibrium. From this consideration he deduced that the area of the parabolic segment was two-thirds that of the rectangle.

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Bishop (1991) noted that on the problem of the objects of this geometric study, that they are not the details of particular shapes but highly general operations and relationships. That, however, this is not to say that objects such as triangles, circles, quadrilaterals and polygons generally are ignored. That, in fact parallels relate to translation, circles relate to rotation and perpendiculars relate to reflection and many Euclidean theorems can be interpreted transformational. That the transformations predominantly studied in schools now are the rigid motion of plan-isometrics-where distances between points are preserved, that is translations, reflections, rotations, enlargements and similarities, and affine transformation, where parallelism is preserved, that is, shears, stretches.

The transformation approach to geometry enables it to be connected with other parts of mathematics like vector, matrices, abstract algebra and group theory. Also, the transformation approach helps to unify mathematics of algebra and arithmetic in the concept of application of vector space and linear algebra. Furthermore the transformation approach can lay the foundation for scientific and technological advancement in that it provides powerful link between geometry, algebra and the physical world. Situations in the physical sciences which involve one or other of these self-same operations; reflection, rotation, translation and enlargement require transformation geometry. Also technology requires these self-same operations in that the mechanisms of technology rely on the principles of reflection, rotation, translation and enlargement.

Interesting aspect of Mathematics education as a related comprehension relies heavily on ‘Transformation Geometry’, a fundamental concept that fosters spatial reasoning, visualization, and problem-solving skills in learners. Despite its importance, learners often struggle to comprehend the abstract concepts and principles underlying Transformation Geometry, resulting in poor performance. Identifying the specific difficulties learners encounter is crucial in developing targeted instructional strategies to address these challenges. Van Hiele’s model, a theoretical framework that describes the levels of geometric thinking, offers a valuable tool for examining learners’ understanding of Transformation Geometry.

In the current 2013 revised Zambian curriculum, school mathematics incorporates transformational geometry. This strand allows learners to make connection across other geometries with the space, shape and measurement learning outcome as well as with algebra and trigonometry. These connections are intended to create more integrated for list knowledge of mathematics (DOE, 2008) and to allow for more nozzle interpretations of the mathematics lessons of Transformational Geometry in the other strands.

One major goal of secondary school geometry is the development of mathematical reasoning abilities and the promotion of a deeper awareness of the real world (The National Council of Teachers of Mathematics (NCTM, 1989). To help pupils achieve this goal, the NCTM suggests that reasoning about shapes should use coordinate and transformation techniques as well as the traditional synthetic techniques such as flip, turn and sliding of an object.

In Zambia, the secondary school educational policy entails five years secondary school study. During this period, pupils are required at the second year (for grade 9s) and fifth year (for grade 12s) of study to sit for an internal examination. At both phases of learning, mathematics is one of the examinable subjects and it is compulsory for all grade 9s, also known as the Junior Certificates Examination and is also compulsory for all grade 12s known as the senior school certificate examination to some extent even for pupils aspiring for a further study in the sciences. However, at both levels of study the mathematics syllabus includes topics drawn from Algebra, Trigonometry, and Geometry among others.

Basing on the content analysis in Zambian context, it is identified that the examiners', findings indicated that the majority of students performed well in questions related to statistics and geometric graphs, but struggled with questions concerning transformations, trigonometry, and earth geometry. Additionally, ECZ stated that 'Transformation Geometry' was the geometry topic where pupils showed lower performance, in comparison to other

Geometry topics. Still, the primary objective of high school geometry is to cultivate in students the ability to think mathematically and effectively tackle real world issues by staying attuned to their surroundings. This implies that students may not have sufficient development of geometric abilities, such as visualizing, rotating, sliding, reflecting, enlarging, shearing, and stretching an object which are crucial for understanding Transformation Geometry and other geometric ideas. Van Hiele's study can help explain and understand the difficulties students face in Enlargement Transformation Geometry. Van Hiele's study was influenced by Piaget, who had previously established four stages of cognitive growth known as the sensorimotor stage, preoperational stage, concrete operational stage, and formal operational stage.

Van Hiele's Model mainly concentrated on five stages of geometric understanding: visualization, analysis, abstraction, deduction, and rigor. (Van Hiele) proposed that learners progress through various stages of geometric thinking. Van Hiele's discoveries on geometric conceptualization were utilized to clarify why numerous high school students were facing challenges in learning geometry. Literature also showed that the levels of geometric understanding in Van Hiele's model were an effective way to assess students' preparedness for formal geometry lessons

These topics are integrated and taught simultaneously every year (Zambia Secondary School National curriculum, 2010). However, indication emanating from the report compiled by the Examination council Zambia (ECZ, 2007, 2008 and 2009) revealed that pupils' performance in geometry and in particular transformation geometry of shear was very poor as compared to other areas of mathematics. The average performance was below 20 percent in, 2004, 2005, 2006, 2007, and 2009(ECZ, 2004, 2009). This evidence leaves much to be desired.

The report pointed to the fact that pupils may lack enough acquisition of geometric skills such as, the ability to imagine, rotate, and twist an object which are essential in facilitating the learning of transformation geometry and other geometrical concepts.

A good teacher has clearly in mind the objectives to be obtained by the pupils. The teaching method or procedure of obtaining these objectives determines the success or failure of the teacher. A teacher of mathematics needs more than a blackboard, a piece of chalk, and a string to draw circles. A teacher must create or stimulate interest and effort, raise questions to be answered by the students, and guide students in establishing good study habits. Various techniques are used by good teachers to make teaching more effective. This includes the use of films, models, manipulative devices, naturally the blackboard, and the bulletin board.

One of the first concepts a child learns pertaining to mathematics is the concept of the counting numbers~ He first learns 1-rords such as one, two, and three. These words are associated with concrete objects. Then, follows the association of one object, the word one, and the symbol 1. Thus, his concept grows from the concrete object to abstract numbers. Supplementary aids can be used to enrichment this pattern of learning.

According to chief examiner report, (ECZ, 2024) stated that," most learners do not attempt to answer the questions on transformation geometry in an exam and those who try they don't always give the required solution and according to (ECZ, 2025) in the

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report published recently the chief examiner further said that this problem has continued to the learners including to those who are rewriting for general certificate examination (GCE) in the article published on Daily mail dated $20th$ October 2025, on the GCE results

The purpose of this paper is therefore to study on the skills and methods on teaching of transformation geometry in secondary schools in an effort to make the teaching of mathematics more effective

2. Statement of the problem

Transformational Geometry plays an important role in solving human problems. It encourages learners to investigate geometric ideas through informal and intuitive approach sensitivity, conjecturing transformation and inquisitiveness. Transformational geometry can lead pupils to the exploration of abstract mathematical concepts of congruency, symmetry, similarity and parallelism, enriching pupil's geometric experience, thought and imagination and thereby enhance their spatial abilities (Guven, 2012).

According to Yanik (2014), Pupils "may discover patterns, construct generalizations, and develop spatial competencies and critical thinking through studying geometric transformation"

In Zambia, geometric transformation among other topics in mathematics as recorded a high number of poor performance. As compared to other geometric topics, transformation geometry is the most topics in which pupils perform poorly (ECZ, 2023).

Many studies have been done regarding the difficulties pupils face in learning transformational geometry, in an effort to provide better ways of ensuring that the challenges pupils face can be done away with, but the problem seems not to cease. This research paper, attempts to add more knowledge to the existing body of knowledge on the study on skills and methods on teaching of transformation geometry in secondary school to make the teaching of mathematics more effective.

3. Purpose of the study

The purpose of this study is to study on skill and methods on teaching of transformation geometry in secondary school to make the teaching of mathematics more effective. This study aims to identify the difficulties of the student faced when learning enlargement of shapes through the procedures of transformation geometry and with a focus on the Van Hiele Model of geometry learning. More emphasis is given to the student's comprehensibility and adaptability to the features of 'Transformation Geometry'

4. Research objectives

- To explore the skills and methods on teaching transformation geometry by teachers.
- To determine the skills and methods applied in teaching transformation geometry.
- To find out how learners apply the skills in answering transformation geometry problems.

5. Research questions

- How do teachers explore the skills and methods in teaching transformational geometry?
- How do the skills and methods applied in teaching transformational geometry?
- How do learners apply the skills in answering transformational geometry problem?

6. Significance of the study

Geometric transformations including translation, rotation, reflection, and dilation are the subject of transformation geometry that encompasses an important area of Mathematics. In order to assist educators in creating successful teaching methods and enhancing students' learning results in designing a crucial area of Mathematics of greater relevance to the study of transformation geometry. Nevertheless, the study intends to investigate these issues in detail.

The abstract nature of the concepts involved in transformation geometry is one of the key challenges that students face when learning it. Pupils frequently have trouble picturing and comprehending these changes, which causes misunderstandings and makes problem-solving challenging. Through the identification and resolution of these conceptual obstacles, teachers can assist students in building a strong foundation in transformation geometry.

One of the biggest obstacles that students have when studying Transformation Geometry is the dearth of practical applications and significance. By incorporating hands-on activities and real world scenarios into the curriculum, teachers can increase students' motivation and comprehension of Transformation Geometry by making the material more interesting and meaningful

The result of the study and its recommendation will provide useful information for teachers, society, school administrator and curriculum developers on how to ease pupils' difficulties in the teaching and learning of transformation geometry in the classroom.

This study will make contributions to mathematics education by way of providing information with regards to the pupils' and teachers in the study on the skills and methods in teaching and learning transformation geometry. It will also contribute to the improvement of policies that addresses the skills in learning transformation geometry. A valuable contribution will be made from this study as the outcome may suggest better and more improved teaching strategies that could help the teacher towards a more effective teaching of transformation geometry in schools.

The findings of this study will help teachers of mathematics in understanding the skills and methods that will be applied in teaching transformational geometry. It will also help them to know the impact of skills in the teaching and learning of transformational geometry and how they can utilize that in making teaching and learning effective.

7. Scope and Limitations of the study

The research was carried in Chongwe district at Mulalika Secondary school in Zambia. The study looks into the obstacles that students have when learning transformation geometry, such as the abstract character of the ideas, the death of practical applications, and problems with spatial reasoning. The research design is based on surveys, interviews, and in-person observations of students and teachers and that will be used in this paper in order to learn more about the main issues that students experience and thereby to find workable solutions. The study's ultimate objective is to identify the fundamental difficulties that students have when learning transformation geometry and to offer instructors useful suggestions for improving their students' spatial domain geometry learning experiences. The purposive deliberation of this paper is to enhance the skills in teaching and learning of transformational geometry.

The study was limited in the following ways:

The time given for the study was not sufficient enough. The financial problem, the research was not funded so there was a shortage of finances. The gargets used required power now due to load shedding work was done in the night as power goes for 12 hours in a day.

8. Theoretical Framework

Theoretical framework is a collection of interrelated ideas based on theories and it attempts to clarify why things are the way they are based on theories.

The study's results were described and explained using the Van Hiele model. According to (Corley), students' preparedness for formal geometry instruction can be accurately assessed using the geometry comprehension levels found in Van Hiele's model. Based on Piaget's work, Van Hiele's research mainly examined five stages of geometric conceptualization. In his book 'Teaching Children Mathematics', Van Hiele proposed that students progressed through different levels of thinking when studying geometry.

The five levels of comprehension of geometry is identified in Van Hiele's model, which are as follows:

Level 1 - Visualization: Illustration at this level, students concentrate on the appearance and spatial relationships of shapes as well as other visual properties. They may not comprehend the properties or relationships of shapes, but they are able to identify them based only on their visual characteristics. It's possible that students won't be able to recognize shapes with varying sizes or orientations.

Level 2 - Analysis: At this level, students start to examine attributes of shapes such as symmetry, angles, and number of sides. They are able to recognize and explain the fundamental characteristics of shapes, such as how many sides a polygon has or how many right angles it contains. Although they may still rely on visual cues, students are beginning to grasp shapes in a more analytical way.

Level 3 - Abstraction: At this level, students are able to comprehend the relationships between shapes and abstract geometric concepts from specific examples. They are able to define shapes in terms of their characteristics and comprehend the relationships between these characteristics. Pupils start to grasp geometry in a more formal and abstract way, defending their conclusions with definitions and theorems.

Level 4 - Deduction: At this level, students are able to solve problems and prove geometric theorems through deductive reasoning and reasoning. They are able to

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comprehend the connections between various geometric concepts and apply geometric concepts to solve challenging problems. At this level, students can use their deep understanding of geometric structures to support their arguments.

Level 5 - Rigor: Pupils at this level can solve difficult, abstract problems because they have a thorough and formal understanding of geometric concepts. Higher-dimensional spaces and non-Euclidean geometries are examples of sophisticated geometric ideas that they can comprehend and manipulate. Hence the challenges experienced by pupils in enlargement in transformation geometry could be explained and understood better by reflecting on the research work done by Van Hiele based on the sequential stages of geometry learning. This sequenced analysis, as featured by Van Hiele has given an extended initiative to the plan of work as adapted by teachers of Mathematics, to make teaching of transformation geometry to the students interesting and motivating.

9. Summary of Chapter One

In order to investigate and describe the problems which were highlighted in the research questions, this research was conducted with all the Teachers in the department of mathematics and all the Grade 12 learners and learners who re-sit for general certificate examination (GCE) at Mulalika secondary school with the purpose of study of skill and methods in teaching as well as investigating and describing the difficulties learners have in transformation geometry. This chapter provided a summary of the study by providing and discussing issues relating to the background and overview of the study, the purpose of the study, the research questions and the significance of the study. The next chapter presents the review of related literature.

10. Definition of terms

The following terms are used throughout the research report and they are defined here to establish a clearer and concise meaning.

Mathematics: This simply implies the study of measurement, number and quantities.

Concepts: General idea about something, this involves understanding the components of a phenomenon.

Creativity: In an attempt to define creativity we look at personality trait of creative individuals. Such individual are always thinking, are always prepared to listen to others opinion, are critical of their work, are analytic and original; have adaptive flexibility,

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spontaneous flexibility, word fluency, the capacity to puzzled, they are motivated, confident, intellectually persistent and moral communication to work.

Curriculum: Planned experiences offered to the learner under the guidance of the school.

Syllabus: is a document containing content which learners are expected to know before being examined.
Manipulative: Concrete material, e.g. Tangrams, Geoboards and Pattern blocks.

Geometry education: Geometry education concern itself with theories, principles and methodologies in the teaching of geometry (Freudenthal, 1971).
Shear: is a subtopic under transformational geometry.

Visual aid: is device or a tool that fosters leaners' visualization

II. Literature Review

Introduction
Literature review gives the pretext and the aptness for the expressive ideology of this paper with supportive identities as taken to reference. It deals with the research reliance that states on students' comprehension and acquisition of transformation geometry is scarce (Bansilal and Naidoo). Accordingly, (Edwards) clarified that 'Transformation Geometry' gave students a chance to hone their geometrical reasoning and spatial visualization skills.

Transformation Geometry is a subset of geometry where students learn to recognize and depict the movement of shapes, according to (Ilaslan), Transformation Geometry is an engaging method of teaching geometry that combines traditional methods and technology use with practical exercises involving real-world objects.

This chapter gives a review of other studies related to the investigation of the difficulties pupils have in the leaning of shear in transformation geometry and the impact of visual aid in the learning and teaching of transformational Geometry.

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1. Geometry

Geometry is a branch of mathematics dealing with the measurement and relationship of lines, angles, surfaces and solids (Fish, 1996). There are many types of geometry found in the field of mathematics. This includes Euclidean, Non-Euclidean, Dynamic, Transformational, Projective, Vectors, Applied and Menstruation geometry.

Early man used geometry concepts to solve everyday problems. They reduced the shape of nature to a less difficult form which was used in a multitude of ways. For example arrowheads, baskets and pottery demonstrated an intuitive knowledge of geometry.

Geometry was utilized by man to build huts, to erect tents and excavate cave (Daffer and Clemens, 1977). As time went on man's knowledge of geometry became more sophisticated. The Egyptian used geometry technique to remark the land that was annually washed away by the Nile River. The word "geometry" meaning earth measure originated as a result of its application by the Egyptian (Greegerg, 1973). Geometry became the science of the study of the measurement of the earth. Thus "the earth consist of structures made up of points, lines and spaces between or on which the lines lie, and the similarity and distinction between the different structures" (Greegerg, 1973:10). It is / on this notion of points and lines, distinction between these lines which they lie on and the congruency between the structures that the Euclidean geometry is based on.

2. Transformational geometry

Transformational geometry is an aspect of geometry which concern itself with the way geometrical shape of objects are transformed into their various image under reflections, translations, rotations, glide reflections and magnifications on a plane. This transformation could be in the form of reflection, translational or rotation depending on the matrix of transformation (Burger, 1992).

Transformation geometry is a dynamic approach to learning geometry in which pupils use hands-on activities with concrete object in addition to using technology (Fuys, Geddes and Tischler, 1985). Rotational transformation transforms the original shape of an object onto its image through a particular angle which could be clockwise or anticlockwise on the plane. It could also be represented by vector or coordinates of the image of the original shape. The image of the rotation is congruent to the original shape. From the above indices one can conclude that Transformations are movements that includes Translation (slides), Rotation (turns) and Reflections through a point half turns or rotation of 180°degree and Reflection of a line 'flip'; (Fuys, Geddes and Tischler, 1985).

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According to Marjoram (1974), researches into how children learn mathematics indicated that learning based on intuition and experiences of the children can enhance retention and application. Also Fremont (1969) noted that these research findings arrived at differing points of view regarding learning process but has established common characteristics as well. That the direction of learning moves from concrete, physical experience with complete freedom to project hypotheses and test them in given situations to the more logical, analytical abstract ordering of what has been done. Furthermore, those teachers who are intended upon building mathematics experience for children can that are all often missed in mathematics classroom.

Lassa (1998; P.6) opined that the children in secondary schools should be made to have pleasure in learning mathematics. He noted that on the whole it used to be dreadful business, and no attempt was made to select subject matters that reflected their culture and be related to their future profession. He further noted that our ideas about the subject depended very much upon the ways in which we study mathematics in the schools and colleges. That some of us experienced only rote learning where teachers showed us on backboard, how to work out a particular problem, then we had to do a number of examples in the same way often without understanding what we were doing, but always trying very hard to get the right answer. However, we did not find it easy to do examples that were not exactly like those we had been shown. And that others of course were fortunate, that teachers helped them to understand what they were doing, by providing practical activities and encouraging them to find out as much as possible for themselves. Thus, that they became confident in dealing with questions they had not seen before. He furthermore noted that with the demand for more mathematical knowledge of all professions and for coping with life there is need to change the method of teaching mathematics in the secondary school.

3. Scope of teaching and learning of transformational geometry

In the teaching and learning of transformation geometry, pupils are expected to carry out tasks involving, Reflection, Translations, Rotation and Enlargement of an object. In doing this, pupils naturally or intuitively solve problems by manipulating concrete object or drawing figures as requested.

In transformation geometry (Rotation) in particular, pupils are expected to find the point, angle, center, symmetry, describe and turn any given figure through a given degree. In doing this the National Council of Teachers of Mathematics (NCTM) such GeoGebra that pupils should construct their own models from straws, make drawing, fold paper cutout, and use mirror to see symmetry (NCTM,1987).

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Fletcher (1976) noted that the first contact pupils in primary schools have with the study of geometry is through motion geometry`: when a shape or an object is moved from one position to another and the motion can be thought of in terms of separate processes, under the guide of the teacher. The movements, she termed transformation and the separate processes involved include turning (rotation), sliding (translation) and mirroring (reflection). She noted that this approach to teaching geometry encourages the pupils to develop geometric thinking, explore and discover geometric ideas to the best of their ability and maturity and eventually aided them in problem solving and stimulated their interest in geometry.

Marjoram (1974) noted that the mostly neglected group of missed opportunities in secondary schools is the development of the study of geometry through the basic transformations of reflection, rotation, translation and enlargement. Bishop (1991;P.860) noted that concerning the methods of teaching geometry, which transformation approach allows, that it can be seen that ideas of rotation and reflection are all intuitively familiar to students. That many actions of body operate on objects in our environment by means of rotation, for example throwing and twisting; and mirrors, windows and water give reflection; sliding and balancing give translation. That this appeals to intuition lays the foundation for geometry as empirical study in schools.

He further noted that actual objects can be transformed as can draw shapes at later stages. That many paper-folding activities can be developed once connections are made with aesthetic ideas of pattern, symmetry and balance. Line symmetry can be approached through the practical methods of folding. That this reinforces the fact that a line of symmetry divides a shape or an object into equal halves each fitting exactly onto the other and can be useful in treatment of similarity in geometry. Tessellation can be looked at and the ideas of congruence are introduced in this framework. Students can be encouraged to look at tessellation in Daniel Adikpe Ekirigwe their world around them. Later the transformation can be symbolized algebraically and systematically analyzed. Combinations of transformations can be explored and suitable axiomatic treatment proposed linking geometry with other branches of mathematics (Wynne, 1977).

The basic aim is to encourage students to approach the study of geometry in a spirit of discovery and adventure for them to understand to the fullest extent consistent with their ability and maturity how mathematical facts are discovered and how mathematical concepts are formulated and can be generalized and extended. In fact, the students should be led to develop mathematical understanding and power. For example

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Archimedes in attempting to find the formula for the area of a segment of a parabola, imagined this area placed at one end of a level arm and the other end the rectangle circumscribing the parabola. He then undertook to determine where the fulcrum must be placed so that the figure may be in equilibrium. From this consideration he deduced that the area of the parabolic segment was two-thirds that of the rectangle.

Bishop (1991) noted that on the problem of the objects of this geometric study, that they are not the details of particular shapes but highly general operations and relationships. That, however, this is not to say that objects such as triangles, circles, quadrilaterals and polygons generally are ignored. That, in fact parallels relate to translation, circles relate to rotation and perpendiculars relate to reflection and many Euclidean theorems can be interpreted transformational. That the transformations predominantly studied in schools now are the rigid motion of plan-isometrics-where distances between points are preserved, that is translations, reflections, rotations, enlargements and similarities, and affine transformation, where parallelism is preserved, that is, shears, stretches.

The transformation approach to geometry enables it to be connected with other parts of mathematics like vector, matrices, abstract algebra and group theory. Also, the transformation approach helps to unify mathematics of algebra and arithmetic in the concept of application of vector space and linear algebra. Furthermore the transformation approach can lay the foundation for scientific and technological advancement in that it provides powerful link between geometry, algebra and the physical world. Situations in the physical sciences which involve one or other of these selfsame operations; reflection, rotation, translation and enlargement require transformation geometry. Also technology requires these self-same operations in that the mechanisms of technology rely on the principles of reflection, rotation, translation and enlargement.

4. Learner’s misconceptions in transformation geometry

Ada & Kurtulus (2010) investigated students’ misconception and errors in transformation geometry. The study analyzed students’ performance in two-dimensional transformation geometry and explored the mistakes made by the students taking the analytical geometry course given by researchers. The subject of the study included 126 third-year students of mathematics in the university. Data were collected from seven exam questions. The result of the analysis showed that these students did not understand how to apply rotational transformation. The mostly mistake observed showed that the students seemed to know the algebraic meaning of translation and also

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of rotation but they did not seem to understand the geometric meaning of these concepts.

In a similar study, Hollebrands (2003) investigated the nature of students understanding of geometric transformations, which included translation, reflections, rotation and dilations, in the context of the technological tool, the Geometer's Sketchpad. The researcher implemented a seven-week instructional unit on geometry transformations within an Honors Geometric class. Students' conceptions of transformations as functions were analyzed.

5. Learning obstacles

Learning obstacles of students to understand the concepts in terms of epistemology in working on the geometry transformation is divided into 5 types, it is in terms of indicators in assessing learning obstacles, namely: a) learning obstacles related to understanding and applying the concept; b) learning obstacles related to visualize the geometry object; c) learning obstacles related to determining principle; d) learning obstacles related to understanding of the problem and e) related to mathematical proofs. According to Noso MN (2019) in the study "Analysis of learning obstacles on transformation Geometry" outlined that the first learning obstacles are students cannot understanding and applying the concept by the command matter. For examples, on the following question: write the definition of a transformation in the field of V (V is Euclid field). Students do not understand the concept; students cannot mention the definition of transformation. It has happened to students with high, medium and low prior mathematical knowledge. Meanwhile, understanding of concepts is very important to be possessed by students, because there is a strong relationship with mathematical reasoning abilities.

6. The Use of Visual aids in the Classroom

Donald Lee Evitt (1958) the use of educational films in the classroom is increasing daily, but few good mathematics films are available. One reason for this is that it is not fully appreciated how films may be put to use in the mathematics classroom. At the present time some people have never seen a film used in a mathematics classroom. However, some very good films have been shown by an educational television station in Chicago. The subjects of the films were beginning geometry and beginning algebra. Each film was thirty minutes long.

The purpose of the film on geometry was primarily one of motivation. This was done by showing how the Egyptians of ancient times used their knowledge of geometry for surveying. Their taxes were calculated according to the area of the farmers' fields. The fanner and tax collector surveyed the fields for the calculation data. However, near the end of the film, projective geometry was introduced (Donald Lee Evitt ,1958).

According Donald Lee Evitt (1958) to justify the use of these films in a classroom he briefly, in compact form, discl1ssed the requirements a good film should fulfill as follows;

- Supply concrete basis for conceptual thinking.
- Have a high degree of interest for students.
- Make learning more permanent.
- Offer experiences which stimulate self-activity.
- Develop a continuity of thought.
- Contribute to the growth and development of the desired concepts.
- Provide experiences not easily obtained in the classroom.

If visual aids have been selected as the medium to enrichen the learning experience, the teacher should utilize the following technique in using these aids to the best advantage. This technique divides itself naturally into three stages: the preparation, the presentation, and follow-ups (Donald Lee Evitt, 1958).

7. Technology in mathematics education

The integration of technology and e-learning in mathematics education has been a topical issue in recent studies. According to the NCTM (2000), technology has become one of the key principles to enhance quality learning in mathematics. As such, learners are expected to develop several competences for them to work effectively in this technology-driven education (Mnguni, 2014). Hoong and Khoh (2003) contend that since “Geometric Transformations” is a visual topic, the utilization of ICT software which aid visualization could have a positive effect on learners’ understanding of the topic. In addition, Idris (2006) identified the causes of difficulties in Geometry learning as visualization abilities and ineffective instruction. According to Vasquez (2015) visual learning is a powerful tool as it involves skills such as observation, recognition, interpretation, perception and self-expression. These skills provide the learner with the opportunity to see, examine something and then to visually recall and interpret information leading to comprehension and understanding of mathematical concepts (Vasquez, 2015).

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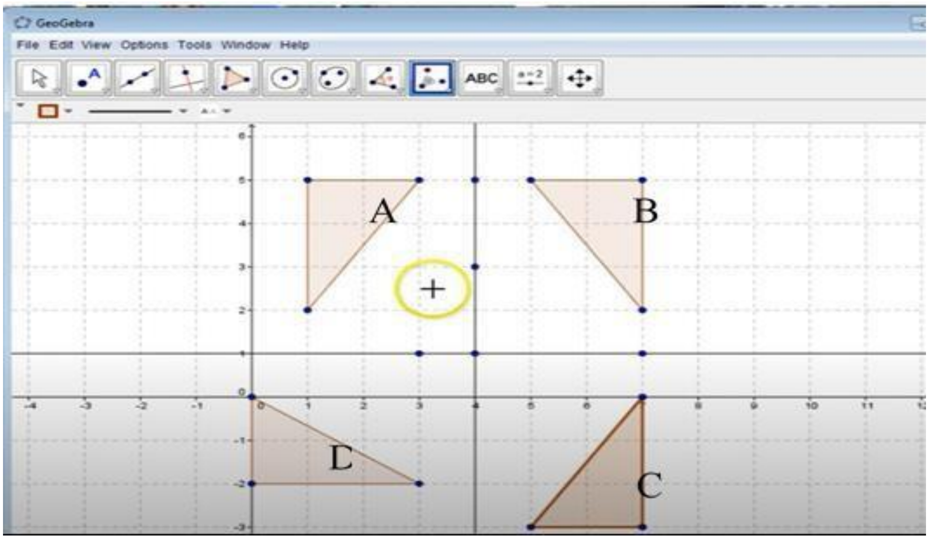

Figure 1: Transformational geometry lesson

8. Using GeoGebra to teach Transformational geometry

According to Bu et al (2011, p15), GeoGebra “is open source and thus is freely available to the international community”, hence Ogwel (2009) encourages the incorporation of the software to teach Geometry. Hohenwarter and Preiner (2007) view GeoGebra as a mathematics flexible tool which enhances visualization of mathematical ideas at all levels of learning. The visualization enhanced by GeoGebra software thus, provides learners with a clear view of both simple and complex mathematical constructions. According to Hohenwarter and Lavicza (2007), GeoGebra software was created to bring learners’ conceptual gaps in understanding geometry and algebra.

This implies that, the creation of GeoGebra was mainly a solution to conceptual challenges faced by learners in geometry and algebra in general. Many studies have demonstrated that GeoGebra is helpful in improving learners’ achievement in learning mathematics (Nazihatulhasanah & Shukor, 2015; Shadaan & Eu, 2013), and that the software develops creativity among mathematically gifted learners (Korenova, 2012).
Dogan (2010) concurs that GeoGebra encourages higher order thinking skills and that it has a positive effect on learners’ retention of knowledge. However, the use of GeoGebra to

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teach mathematics is a challenge to teachers due to lack of training in the use of technology (Davidson, Richardson & Jones, 2014; Alharbi, 2013). The lack of training is due to insufficiency of computers and other necessary devices to implement ICT-integrated lessons (Arienda et al, 2016)

9. Theoretical Framework

The study was guided by the theory of Realistic Mathematics Education (R.M.E) (Freudenthal, 1978; Gravemeijer, Cobb, Bowers, & Whitenack, 2000). Since the study is about the impact of visual aids on learners' performance in Transformational Geometry, it fits well in the tenets of RME because RME is grounded in the realistic connections of mathematical ideas (Bu, Spector & Haciomeroglu, 2011).

According to Bu, Spector & Haciomeroglu (2011), RME considers mathematics learning as a human activity and as a process of guided reformulation of mathematical ideas through horizontal mathematisation. In horizontal mathematisation, Bu et al (2011) assert that real world problem situations are symbolized by mathematical models, which resonates with the topic under study. In short, horizontal mathematisation is simply transforming daily problems to mathematics symbols. Similarly, Clements and Sarama (2013) state that RME includes the development of a model which allows learners to associate problems with contexts, identify relevant mathematical concepts, solve problems, and interpret the solution on the basis of their contexts.

Viewed from the theoretical lenses of RME, GeoGebra is a tool that allows learners to mathematics realistic problem situations, and experiment with some models using multiple representations to formulate abstract mathematical ideas (Spector & Haciomeroglu, 2011). The visual model, which is demonstrated in GeoGebra, can generate problem solving skills in the learners, which they can use to solve realistic problem situations. Laurens et al (2017) contends that RME starts with problems relevant to learners' experiences and knowledge.

The teacher then facilitates to assist learners to solve the contextual issues. This problem-solving activity breeds positive impact to the learners' cognitive achievement that is related to their understanding of mathematics. This means learners can be best taught mathematics by asking them to mathematically solve problems that they experience on a daily basis, hence, improving their creative thinking. Laurens et al (2017) concurs that mathematics is not only for academics, but is involved in one's

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daily life. Thus, mathematics, according to the RME theoretical framework, constitutes a process where learners mathematically formalize their informal understanding and intuition

III. Research Methodology

Introduction

This chapter presents the justification and essence of the choice of research methods adopted for the study. It is divided into eight (8) sections with the first section discussing the knowledge and values arguments for the choice of research strategy. The sources of data will be discussed in second section, the third section examines the methods of data collection and the fourth is concerned with sampling technique and frame. The methods of data analysis and presentation will be discussed in the fifth section and the sixth section will identifies the ethical considerations.

1. Research Design

A research design can be understood as the framework in which data is collected and analyzed (Bryman, 2008). The study adopted a mixed method design that included both quantitative descriptive design and qualitative case study design. Both methods of investigation were used because they provide the wide-ranging data that resulted in an in-depth understanding of the pupils' thinking processes that involves shear and stretch problem solving situation. The Qualitative aspect of the research used the interview while the quantitative method was in form of the test. To further maintain validity, triangulation was used to confirm and compare result from these two research instruments.

2. Source of Data

Both primary and secondary data sources were used for this study. The primary data was collected through physical visitation of the study area in two weeks field survey in the area of study. Furthermore, the secondary data was collected from document review, magazines and various articles

3. Methods of Data Collection /Research Instruments

This section shows the methods of data collection / research instruments that were employed during data collection. The methods that were used include the following:

Semi-Structured Interviewing

In this study, a semi- structured interviewing was used for the collection of data. The key informants that were used are the head of department mathematics and teachers for mathematics. Different interview guides were used for various informants but the content that was used in order to answer the research questions. Through respondents were given exactly the same context of questioning (both closed and opened ended questions inclusive) so that the replies could be aggregated, semi- structured interviews by nature are mostly flexible and this was giving the interviewees a great deal of leeway in how to reply (Bryman, 2008, 438). It was also offer an interviewer the opportunity to probe further and to clarify issues of relevance to the study.

Interview data was obtained from a group of six teachers. The interview was administered immediately after school hours for duration of two hours a day and it lasted for three days. The purpose of the interviews was mainly diagnostic. The content of the interview was on how long has a teacher save in service teaching mathematics ,the methods ,approach and strategy he/she use in the teaching of shear and stretch (see Appendix B). The interview method was adopted so as to check on teachers how they handle the topic and types of methods to compare with response from learners. The interviews were audio-taped in addition to pencil and paper responses.

Classroom Observations

This is a systematically way of collecting data by watching what happens in a classroom setting (teaching, student behavior, interaction patterns, learning activities) and recording it using agreed procedures. It is mainly used to understand processes (how learning happens) rather than only outcomes. The researcher observed pupils during transformation geometry lessons to gather data on their interactions, behaviors, and problem-solving strategies.

The researcher targeted mainly to observe the following include:

- Teacher’s practices: questioning style, explanation methods, feedback, and classroom management.
- Student behaviors: participation, engagement, collaboration, attention, asking questions
- Interaction patterns: teacher-student talk, student-student talk, group work dynamics.
- Learning activities: task types, pacing, use of materials/resources.
- Classroom environment: inclusivity, routines, learner support, behavior norms

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The researcher used structured observation, is a type of classroom observation where the researcher uses a pre-planned observation schedule (checklist or rating scale) with specific categories and indicators to record what happens in the classroom. Data is usually collected in a systematic, consistent way across lessons, classes, or teachers.

4. Study population

The study targeted the population of sixty six (66) that were selected and it comprised of six (6) teachers from the department of mathematics and sixty pupils from Mulalika secondary school that comprises thirty (30) from academic class (pure class) and thirty (30) from vocational class. The rational is that all the above respondents were stakeholders. Target population refers to the cumulative elements of study from an environment in which information is gathered from.

5. Sampling Frame and Technique

Both the stratified random and purposive sampling techniques were used because of the mixed research strategy adopted. The stratified random sampling was used to guide the selection of appropriate sample to ensure that, generalization of sample findings are representative of the population which is a key characteristics of all probability sampling techniques (Bryman, 2008). The stratification was also based on gender (male and female strata) and age.

The other technique that was adopted is purposive sampling which Bryman (2008) argues that it is used to select subjects based on their relationship with the research questions. This technique was used to identify key informants like the head of department mathematics and teachers.

The sampling frame consists of six (6) members of department of mathematics at Mulalika secondary school. In addition, a total of sixty (60) pupils participating from Mulalika secondary these were sampled and this consist thirty (30) from academic class (pure class), other thirty (30) from a vocational class. The choice of these classes was influenced by evidences of some of the explanatory variables of pupils

6. Reliability and Validity

Reliability

The reliability of a test or instrument refers to the extent to which it consistently measures what it is supposed to measure (Creswell, 2010). A test is reliable to the degree that it measures accurately and consistently, yielding comparable results when administered a number of times (Agwubike & Momoh, 1995). To ensure reliability of the data collected in this study, the contents of the written test and interview went through verification from an independent body (colleague) that is knowledgeable in the line of mathematics education to ascertain the degree to which the contents of the test items and interview were in harmony with the intended purpose.

Validity

Validity refers to the extent to which inferences made on the basis of numerical scores are appropriate, meaningful and useful to the sample (McMillan and Schumacher, 2001). Validity also checks whether the instruments provides an adequate sample of items that represent that concept (De Vos et al, 2003).

In this study, both construct and content validity was used in this study to check if the observation and interview questions really measured the concepts that was assumed it measured. Also if they correlated with the van Hiele's level of learning geometry transformation. In doing this, some tasks in the observation sheet were chosen for the test on the basis that they have been previously used by soon (1992) to describe student's levels of achievements using the van Hiele levels of learning in transformation geometry. The criteria for the selection of questions from textbooks were based on the questions ability to solicit learners' to visualize, identify, analyze and describe shear and stretch geometry. Furthermore, all other aspects involved in the investigations which were, assigning learners to the van Hiele's levels, analysis and interpretation of data, we gain solicited review, feedback and critique of work from professional colleagues, and our supervisor. These steps were included in the process in ensuring that the research findings are meaningful and reflected learners' perceptions.

7. Data Analysis and Presentation

The quantitative data was analyzed statistically using basic techniques for descriptive and inferential statistics and the qualitative data was analyzed by describing the unit in detail.

The raw data was first be coded and categorizing the variables into the appropriate types namely interval, ratio, ordinal, nominal and dichotomous variables (Bryman, 2008, p.321). The categorization of the variables will help to determine the appropriate quantitative data analysis technique which should be used where necessary

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8. Ethical Consideration

The basic understanding of ethics concerns acceptable and unacceptable standards of human behavior and their environment. The information or the data collected will be treated confidentially for academic purposes. In the classroom observation, learners did not write their names on the answer booklet. On the aspect of an interview, every response from the respondents will be treated with significance as this will help to address the challenges in the delivery of transformation geometry to learners.

The researcher also informed all the participants that the information that he is going to collect from them was used for educational purposes and he maintained confidentiality at all times. This is evident in the manner the questionnaires were designed. There was total anonymity as no names were required when filling in the observation sheet.

9. Summary for chapter three

This chapter described the research methodology of the investigation that was conducted in Mulalika secondary school in Chongwe District Zambia. A detailed description of the study samples, instruments used and the validity and reliability of these instruments were discussed. The chapter further described the sampling, data collection methods and analysis. The next chapter presents the analysis of data collected from the classroom observation and structured interview

IV. Analysis and Interpretation of Data

Introduction

This chapter presents the analysis and interpretation of data obtained from the classroom observation and the structured interview administered to learners and teachers. The aim of the study was to study on skill and methods on teaching of transformation geometry in secondary school to make the teaching of mathematics more effective. A detailed analysis and discussion of findings of the classroom observation and structured interviews are presented in this section.

The three research questions that guided the study are

- How do teachers explore the skills and methods in teaching transformational geometry?
- How do the skills and methods applied in teaching transformational geometry?
- How do learners apply the skills in answering transformational geometry problem?

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1. Presentation and Discussion of Findings

Introduction

The findings are presented and discussed according to the three research questions stated in above. Firstly, a table showing a general description of teachers in the classroom observation and their exploration of skills and methods in teaching transformational geometry are provided in Tables. This is followed by the discussion of teachers during classroom delivery, the skills and methods on teaching transformation geometry.

Classroom observation on teachers

The study demonstrated Demographic Information of Teachers, Teaching Experience, Teaching Methods Used; Skills Required in Teaching Transformation Geometry, Teachers identified the following important skills, and Challenges Faced by Teachers

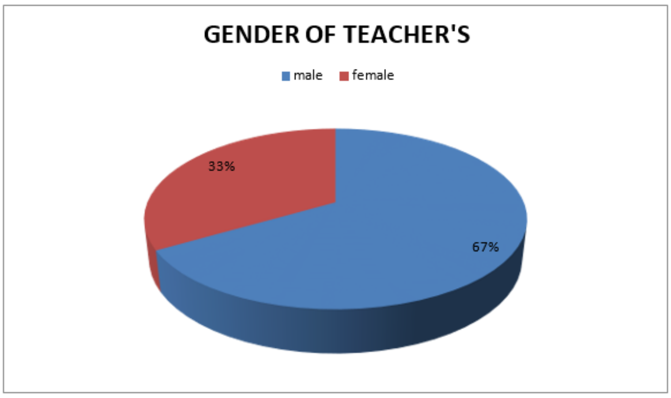
Demographic Information of Teachers

Table 1 Gender of Teachers

Gender	Frequency	Percentage (%)
Male	4	67
Female	2	33
Total	6	100

The findings indicate that most mathematics teachers were male.

The gender of teachers can be summarized in a pie chart 4.1



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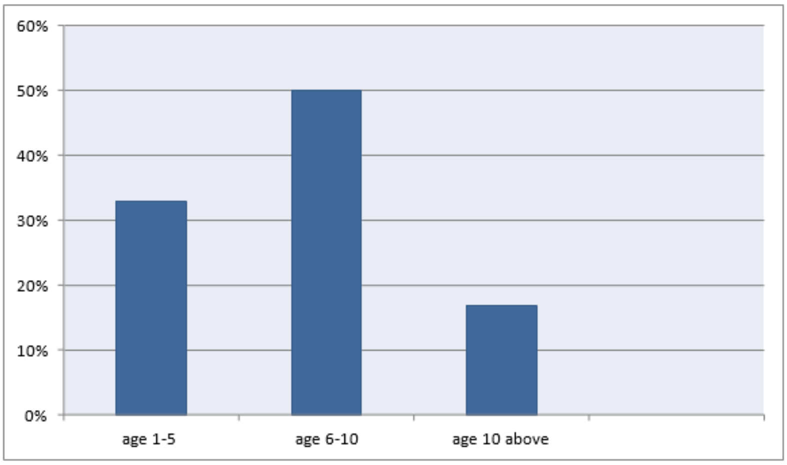
Teaching Experience

Table 2 shows the teaching experience of teachers

Years	Frequency	Percentages (%)
1-5	2	33
6-10	3	50
ABOVE 10	1	17
Total	6	100

Most teachers had between 6 and 10 years of teaching experience. The teaching experience of teachers can be summarized by bar chart



Question 1

Teaching Methods Used Table 4.3

The table below shows the teaching method used in teaching transformation geometry

Methods	Frequency	Percentages (%)
Demonstration	5	83
Group Discussion	4	67
Question and Answer	6	100
Discovery Learning	3	50
ICT Integration	2	33

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The findings show that the question-and-answer method and demonstrations were the most commonly used teaching methods.

Question 2

Skills Required in Teaching Transformation Geometry Table 4.4
Teachers identified the following important skills

Skills	Frequency	Percentage (%)
Knowledge of geometric concepts	6	100
Ability to use teaching aids	5	83
Communication skills	5	83
Classroom management	4	67
ICT skills	3	50

These findings suggest that both content knowledge and pedagogical skills are important for effective teaching.

Challenges Faced by Teachers Table 4.5

The Table below shows the Major challenges that teachers face in delivering transformation geometry included:

Challenges	Frequency	Percentages (%)
Lack of teaching materials	4	67
Limited ICT facilities	3	50
Large class sizes	6	100
Learners' poor mathematical background	6	100

These challenges negatively affect the teaching of transformation geometry most teachers fail to deliver the topic looking at the class sizes, the poor learner's backgrounds in mathematics, the teaching materials and limited ICT facilities.

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Question 3

Classroom Observation on learners

The study shows how learners were responding during class session to methods, skills and their application in real life related today in daily life of transformation geometry.

Methods that make learners effective participate in class Table 4.6 Most learners reported that:

Methods	Reasoning	Frequency	Percentages (%)
Demonstrations	improved understanding	50	83
Group work	increased participation	55	92
Practical activities	activities made geometry easier	60	100
ICT tools	ICT lessons were interesting but rarely used	56	93

The above methods were used in classroom observation, learners were able to grasp concept mainly on practical activities, group work and they were interactive. On the use of ICT tools brings more confidence on the sense of learners and demonstration allow learners to have more understanding in the learning of transformation geometry.

4.1.3.2 Application of skills in answering the transformation Geometry by learners

Table 4.7. Most learners reported that:

Skills	Reasoning	Percentage (%)
I.C.T skills	Making the transformation lesson so meaningful	80
Practical activities	Learners fluently interact	95
Visual aids	Able to see the real images	
Communication and collaboration	Able to communicate and collaborate with language in transformation	100
Critical thinking and creativeness	Allow to think and be able to come up with new things	75

The study looked at skills that are used by the learners in answering the transformational geometry problems, learners are able to communicate and collaborate in transformational language e.i mapped onto, the image and the object, the vertices. Practical activities allow learners to interact real object materials. I.C.T integration

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lesson make the topic so meaningful that allow the learners to be able to handle transformation problems very faster and easily.

The study found that teachers predominantly use learner-centered methods such as demonstrations, discussions, and question-and-answer techniques. These methods improve learner participation and conceptual understanding of transformation geometry. However, inadequate teaching resources, overcrowded classrooms, and limited ICT facilities reduce the effectiveness of instruction. These findings are consistent with research showing that active learning approaches and sufficient instructional resources improve mathematics learning outcomes.

The analysis above has shown that senior secondary learners have serious problems in transformational geometry. Table 4.6 above demonstrated that learners' have problems with higher order questions and the use of I.C.T tools. As the level of difficulty increased, the number of learner's getting wrong answers also increased. Table 4.6 further demonstrated the existence of the problem by pointing out specifically on the Van Hiele levels where learners' challenges are in terms of transformational geometry. According to table 4.6, the learners have challenges in level 2, 3 and 4. In other words, the learners have problems with questions that demand analysis and deduction. As this was not enough the comparative analysis with the Van Hiele degree s of acquisition illustrated in table 9 showed remarkable differences in terms of learners' levels of acquisition. The data revealed that learners' who participated in the study had low degrees of acquisition which culminated in learners failing to answer or partially answer higher order questions.

Table 4. 8

Table summarizing the performance of students on Van Hiele's Degrees of Acquisition

Level Trype	No	Low	Intermidiate	High	Complet
	Aquisition	Aquisition	Acquisition	Aquisition	Aquisition
1	0	4	18	44	4
2	10	33	14	13	0
3	25	15	16	14	0
4	13	34	12	9	2

Table 4.8 above gives a clear picture of the students in this study. The data showed that there were more students who had attained either no acquisition or just low acquisition in level 2, 3 and 4. For instance, there were 43 learners in level 2 who had acquired either no acquisition or low acquisition as compared to 27 learners who had acquired either intermediate or high acquisition degrees. This means that the students were more comfortable with basic questions of level 2 types than those which require them to analyze situations. For the observation used in this study, many learners failed to find the image of a figure using the matrix. In level 3 and level 4 the picture was a slit different. For example, there were 40 learners who had acquired either no acquisition or low acquisition at level 3 as compared to 30 students who had attained intermediate or high acquisition. In a nutshell, we can conclude from the data above that almost all the students who took part in the study have not acquired geometrical skills to enable them answer questions at higher level. Most of them are still operating at the first two Van Hiele's degrees of acquisition.

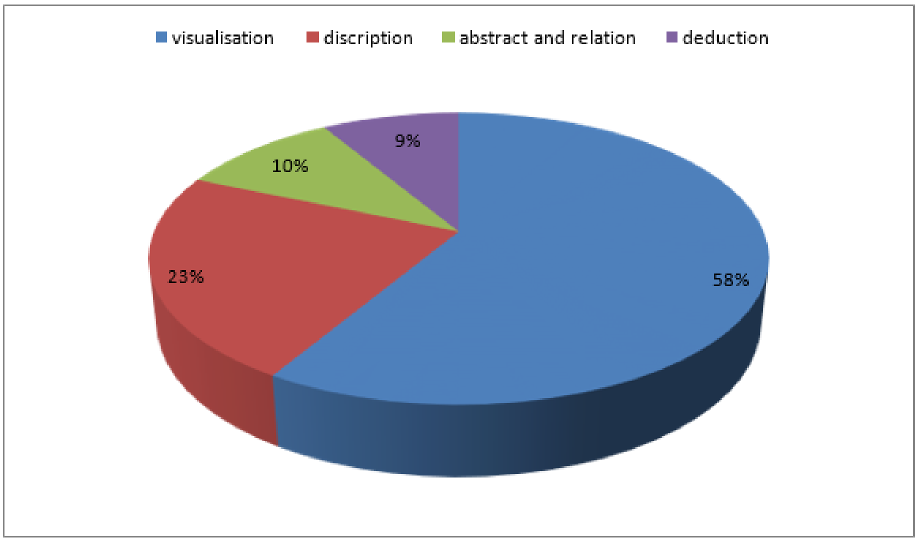

Figure 2: Pie chart showing learners areas of difficulties

The above pie chart is derived from the identified learners' area of difficulties in Table 4.8. It illustrated the various area learners are having difficulties

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Question 1

Structured interviews for teachers

This was on approach, methods and strategy that teachers use in teaching the topic of transformation geometry. This was done through interview; six (6) teachers were interviewed who have taught this topic for so many years with different qualification and different years of serving in teaching services. Their responses will be summarized below;

Interviewer: have you ever taught transformation geometry?

Teacher's: yes

Interviewer: what approach and methods did use to teach pupils transformation in your class?

Teacher (TS01M): The approaches I used by giving my pupils more work to research on before I start teaching transformation. These are given in groups of four (4) thus group work and give them encouragement toward the learning of mathematics as well as the benefit of each topic.

Teacher (TS02M): The methods I use to teach this topic is demonstration, I give a question and solve as well as showing them how to go about it, the second question I give learners to solve and clarify to the answer given.

Interviewer: what challenge do you encounter when teaching this topic of transformation and what some of challenges that most pupils experience in learning of this topic?

Teacher (TS03F): There are several number of challenges that us teacher's encounter in teaching this topic of transformation such as in inadequate teaching and learning resources for example text books are not enough, this also applies to learners they don't have text book to research from. Overcrowded of classroom, teaching of this topic requires interaction between teacher and pupils, more work in topic requires more of demonstration. Pupils lack interest in topic as the nature of the topic requires the use of the graphs.

Interviewer: how did you help learner to overcome the challenge in topic?

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Teacher (TS03F): I emphasis the pupils to practice more questions to go through and consult were they not clear due to technology they can also search on You tubes to more knowledge and also to work in groups. The other thing was encouragement to have interest in each topics of mathematics.

Interviewer: from your own experience, what you say about this topic (transformation).

Teacher (TS04F, TS05M and TS06): Transformation geometry is an interesting topic in mathematics that contribute to the classification of mathematics as a practical subject. It is a topic that requires more attention and it teaching requires connecting concept. It has been years now our department has been having poor performance on results of learners.

The above responses from teachers provided evidence that some teachers and learners have challenges in learning and teaching of transformation that require more attention. The symbols TS01M represent teacher number one male to be interviewed and TS03F teacher number 3 female according to their responds. The approaches and methods teachers are using have some challenges; this topic transformation is a practical topic that needs to be explaining using real things connected to the daily life activities. The teaching should more practical than theory to help learners to have interest in this topic and encouragement on studying using different material.

2. Analysis of results Table 4.9

(a) Analysis of vacation class results in terms of Mean and Standard Deviation

Level	Number of participant	minimum	maximum	mean	Standard deviation
1	30	33	100	68	16
2	30	4	69	27	16
3	30	3	83	25	18
4	30	0	87	31	27
Total no	30				

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Table 4.9 summarized how vacation class learners faired on the four Van Hiele levels. From the table above we noted that vacation learners performed very well at level 1 where the mean was above 60%. Here, the minimum average score was 33% compared to the maximum score of 100%. Nonetheless, learners' performance on the next three levels was pathetic and below average. For example, the group average in level 2 was only 26.6% or approximately 27%, while for level 3 the group average reduced to 24.9% or approximately 25% and was only at 31% at level 4. Worse still, the group minimum score at level 4 was 0% while the maximum score was 87%.

Table 4.10: Analysis of academic class results in terms of Mean and Standard Deviation

Level	Number of participant(N)	minimum	maximum	mean	Standard deviation
Level 1	30	50	89	65	13
Level 2	30	4	64	27	16
Level 3	30	3	37	15	9
Level 4	30	0	47	30	10
Total no	30				

Table 4.10 is a summary of how academic class faired on the Van Hiele model. As illustrated in the table above; academic class, like the vacation class did extremely well on level 1. However, students' performance at levels 2, 3 and 4 was not impressive. The mean score for these levels were 27%, 15% and 30% respectively. The minimum scores were 4%, 3% and 0% whereas the maximum scores were 64%, 37% and 47%. These results demonstrated the severity of the problem. The results showed that internal students had problems with levels 2, 3 and 4 like their vacation class.

To further demonstrate that there was no significant difference between academic and vacation class' performance on the Van Hiele model, a t- Test was performed on the two groups at $\alpha = 0.05$ the results are tabulated in table 7 and table 8 below

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Table 4.11: Analysis of vacation class learners' results using One-Sample t-Test

				α =0.05		
Level	T	Df	Sig(2 tailed)	Mean	95% confidence interval of the difference	
				Difference	Lower	upper
1	23	29	0	68		73
2	10	29	0	27	21	32
3	8	29	0	25	18	32
4	6	29	0	31	24	41

Table 4.12: Analysis of academic class learners' results using One-Sample t-Test

				α =0.05		
Level	T	Df	Sig(2 tailed)	mean	95% confidence interval of the difference	
				difference	Lower	upper
1	29	29	0	65	61	70
2	9	29	0	27	21	33
3	10	29	0	15	12	18
4	10	29	0	30	24	36

From the two tables, 4.11 and 4.12 it was noted that no significant differences existed between internal and external' in acquisition of concept vacation learners in transformational geometry. This can be inferred from the means of the two groups which are close to each other. For Instance, the means of the academic class learners on the four levels were 65, 27, 15 and 30. On the other hand, the means for vacation class learners on the four levels were: 68, 27, 25 and 31.Similarly, the t values depicted the same picture. For example, the t values for the academic class learners on the four levels were: 29, 9, 10 and 10; whereas those for vacation class learners were: 23, 10, 8 and 6. So, looking at these sets of values we concluded that no significant differences occurred between internal and external students in acquisition of geometry concepts. While one may be forced to argue that differences do exist, for example if we look at the mean of 68 against that of 65 for vacation class learners and academic class learners in level 1. In the same way if we compare the t value of 29 for academic class learners against the t value of 23 for vacation class learners in level 1, we could see the

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differences. However, here we are interested in significant differences, differences which will impact very much on the performance of the two groups.

3. Summary of the chapter

Discussion and finding

The first objective of this study was aimed, to explore the skills and methods on teaching transformation geometry by teachers According to the study, the study found that teachers predominantly use learner-centered methods such as demonstrations, discussions, and question-and-answer techniques. These methods improve learner participation and conceptual understanding of transformation geometry. However, inadequate teaching resources, overcrowded classrooms, and limited ICT facilities reduce the effectiveness of instruction. These findings are consistent with research showing that active learning approaches and sufficient instructional resources improve mathematics learning outcomes.

Apart from this, learner's demonstrated serious challenges on the finding of the image in the sense that most of them could not use the given matrix sheared to objects. This was clearly seen in question 2 and 3 in the results during observation.

The second objective was aimed to determine the skills and methods applied in teaching transformation geometry. According to this study, teachers showed seriously concern on analyzing the identified skills and their application to overcome problem in transformation geometry. These skills will allow learners to more interest in transformational geometry and mathematics in particular. These skills and methods will explain that transformation is not taught abstract only because learners will only have abstract knowledge, transformational geometry can be easier understood by applying different skills and methods.

Thirdly the objective aimed to find out how learners apply the skills in answering transformation geometry problems, learners are able to communicate and collaborate in transformational language e.i mapped onto, the image and the object, the vertices. Practical activities allow learners to interact real object materials. I.C.T integration lesson make the topic so meaningful that allow the learners to be able to handle transformation problems very faster and easily.

The study also revealed that learners struggled to understand the terminology used in geometry, which prevented them from applying these terminologies when describing figures that were enlarged. The study's final findings indicated that most learners were not exposed to more geometrical problem solving scenarios and lacked sufficient and pertinent prerequisite knowledge for application of the scale for enlargement as denoted in the problems of transformation geometry

V. Conclusion and Recommendation

Introduction

In this chapter, the conclusion will be discussion in section 5.1. This will be followed by the recommendations in section 5.2. The interview and the classroom observation that was conducted with learners were very useful in providing data used in the description of students' difficulties in transformation geometry of shear and stretch.

Conclusion of the Study

The study examined the skills and methods used in teaching transformational geometry within the Zambian secondary school curriculum. Based on the findings, it can be concluded that effective teaching of transformational geometry requires teachers to possess adequate mathematical knowledge, pedagogical skills, and the ability to use appropriate teaching and learning resources.

The study established that methods such as demonstration, discussion, guided discovery, group work, practical activities, and the use of visual representations can make transformational geometry easier for learners to understand. In particular, the use of diagrams, grids, tracing paper, geometric models, and other locally available materials helps learners to visualize transformations such as translation, reflection, rotation, and enlargement.

The findings further indicate that learner-centered approaches are more effective than relying mainly on teacher explanation and memorization. When learners are given opportunities to manipulate shapes, draw figures, investigate patterns, work collaboratively, and explain their reasoning, they are more likely to develop problem-solving, critical-thinking, communication, and spatial-visualization skills. These approaches are consistent with the objectives of the Zambian Competence-Based Curriculum, which emphasizes the development of practical competencies rather than knowledge acquisition alone.

However, the effective teaching of transformational geometry may be affected by challenges such as inadequate teaching and learning materials, limited time, large

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classes, learners' difficulties with spatial concepts, and insufficient opportunities for practical activities. These challenges can limit teachers' ability to employ learner-centered and activity-based methods effectively.

Overall, the study concludes that the quality of teaching transformational geometry in Zambian secondary schools can be improved when teachers combine appropriate pedagogical methods with practical activities, visual aids, technology where available, continuous assessment, and learner participation. Strengthening teachers' professional skills and ensuring the availability of suitable teaching and learning resources are therefore important for improving learners' understanding and performance in transformational geometry.

In conclusion, effective teaching of transformational geometry should move beyond the traditional emphasis on definitions, procedures, and memorization. It should provide learners with meaningful opportunities to explore, construct, transform, discuss, and apply geometric ideas in real-life and mathematical situations. Such an approach has the potential to improve learners' conceptual understanding and support the successful implementation of the Zambian Competence-Based Curriculum.

2. Recommendation of the Study

Having successfully conducted the study, the researcher decided to make the following recommendations arising from the finding of the research. To begin with, one the recommendations is that the education curriculum developers should consider revising or completely developing curricula for teacher training institutions which will assist teachers teach at secondary school level without problems. That is curricula which are relevant to secondary school mathematics unlike the situation now, where would be teachers are empowered with too much content knowledge which in most cases is not beneficial to both the teacher and the pupils. It will be wonderful to see newly created universities and colleges of education take up this challenge.

Secondly, the curriculum developers should also consider bringing forward topics such as transformational geometry, earth geometry and linear programming in order to afford pupils time to learn them in good time as these topics are very important and they are always assessed. Instead of putting these topics in the third term of grade 12, it would be ideal to put them in the first term of grade 12.

Thirdly recommendation is that teacher education in Zambia should consider taking a leading role in monitoring private teacher colleges as they are doing to public colleges

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of education. Teacher education should get involved in academic recruitment of staff, curriculum issues and the quality of teachers being produced in these institutions if education standards especially in mathematics have to be improved. This is so because; the teachers from private teachers colleges are being employed by the government; some of whom are teaching in secondary schools.

This is very important because there are so many private teachers colleges that have mushroomed due to liberalization of the education sector. As such, government through the ministry education should strengthen its monitoring system in order ensure quality of education. Moreover, there is need to continue supporting continuous professional development Programmes or activities in colleges and secondary schools and assisting and supporting teachers of mathematics pursuing further studies.

In fact, the idea of "fast track" Programme for teachers should be continued in order to reduce on the shortage of mathematics university teachers to teach in secondary schools. However, a suitable curriculum should be devised for this Programme so that the teachers being trained have the right or suitable content to teach secondary school mathematics as earlier alluded to.

The fourth recommendation instructors should make an effort to inspire students' interest in the subject and provide them with the background information they need to understand transformation. This should involve exposing students to real-world scenario-based problem-solving exercises that call for explanation as well.

The fifth recommendation the mathematics curriculum in secondary schools should be adapted to the different levels of thinking and should teach students important and useful concepts that will force them to justify and explain their conclusions and develop their critical thinking skills.

Teachers ought to be equipped with cutting-edge methods and resources so that students can grasp transformation geometry concepts with ease through visual aids, interpretations, and actions. This will be beneficial. Suggestions for further investigation can be taken in relational identity to the relevance of geometry and geometrical language in the learning of transformation geometry. This study may give greater impetus to creative designing and plan lay out that can be formulated for the futurity to practice.

Finally the approaches and methods that teachers apply in teaching transformation geometry or geometry in general should be more flexible , realistic , related to real life and more emphasis on practical's that can influence learners to have interest in mathematics and avoiding teaching learners for exams sake. In addition the curriculum developer should added this topic to junior secondary school mathematics in order to be connected with special paper from primary school Grade 5-7 syllabus as pre-requisite knowledge. In junior secondary learners should be learning basics of the transformation geometry.

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